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# A Note on the Selection of Wave Spectra for Design Evaluation

by

Kathryn K. McCreight

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## ABSTRACT

*A brief discussion of issues related to selection of representative wave spectra for use in evaluating the performance of a hullform in a seaway is given. A method for selecting a small number of wave spectra is outlined.*

## ADMINISTRATIVE INFORMATION

This effort was sponsored by the Defense Advanced Research Projects Agency under the Maritime Platform Technologies Program and was managed and performed by the Naval Surface Warfare Center, Carderock Division.

## INTRODUCTION

In evaluating the performance of a floating body, it can be important to determine how well that body can operate in the sea conditions it is likely to encounter. Typically, during the design phase a range of wave spectra are considered using motion prediction programs. When a frequency domain analysis is appropriate, a wide range of spectra can readily be considered. A smaller set is considered for time domain analyses due to time and cost constraints and an even smaller subset is selected for experimental investigations.

When selecting a limited number of wave conditions to use in an evaluation, consideration is given to the expected geographical operational region and to the motion characteristics of the floating body. Databases of wave statistics have been developed which are used to identify characteristics of spectra which might occur in different seasons and geographical locations. Motion characteristics of the floating body determine which of the spectra which are likely to occur are most apt to excite the largest responses in the body. A basic characteristic of a motion response is its natural period, the period at which the body will oscillate following a brief external force and the period associated with the motion transfer function peak. When a response is lightly damped, the motion transfer function will be sharply peaked and its responses to different seaways may vary significantly, depending on the energy distribution of the seaway; however when a response is heavily damped, sensitivity of motion responses to variations in spectral shape is minor. For monohulls, pitch and heave are heavily damped, but roll is lightly damped so that roll responses are sensitive to variations in the wave environments. SWATHs and other floating bodies with relatively small waterplane areas may be lightly damped in all modes of motion. An additional factor is size since natural periods of large bodies are larger than those of similar, smaller bodies. Thus, for very large floating bodies, long period waves such as those present in swells are likely to excite a response.

Thus, the process of selecting wave conditions to study is complex and must include consideration of characteristics of the floating body. The discussion below is not comprehensive; rather, it is limited to presenting information relevant to one approach for design evaluation selection of seaway spectra, including those with swells superimposed on a seaway.

## RESPONSES OF FLOATING BODIES TO WAVES

A floating body's behavior in waves is dependent on the wave environment and its motion characteristics. Some fundamental definitions are briefly described below in order to show the relationship between the environmental excitation and the floating body's responses.

### WAVE SPECTRA

Seaways are often described in terms of "sea state", with the sea state number increasing in some manner with the height of the waves. However, the term sea state provides limited information about the character of the seaway. More information can be obtained by analyzing the wave height time history and transforming it into a wave spectrum, if certain assumptions about the character of the seaway are made.<sup>1</sup> Typically the wave spectra presented are point spectra, that is, consideration is given to the energy content at a point, without regard to the direction from which the waves originated. A wave spectrum can be thought of as representing the distribution of energy as a function of wave frequency, with units of  $\text{m}^2\text{-sec}$  or  $\text{ft}^2\text{-sec}$ . A measure of the severity of a seaway can be calculated by integrating the wave spectrum with respect to wave frequency. Four times the square root of this integrand is referred to as the significant wave height of the seaway,  $H_{1/3}$ , which is assumed to be the average of the highest 1/3 of the wave heights in a time history. The wave frequency (or wave period) associated with the maximum energy is referred to as the modal frequency,  $\omega_m$ , of the wave spectrum, where the units of  $\omega_m$  are rad/sec. The modal period is  $T_0 = 2\pi/\omega_m$ . In Figure 1 three wave spectra with a significant wave height of 1.88 m (6.17 feet) are presented. These represent mid-Sea State 4 spectra and illustrate the variations in energy distribution which occur in nature for the same sea state.

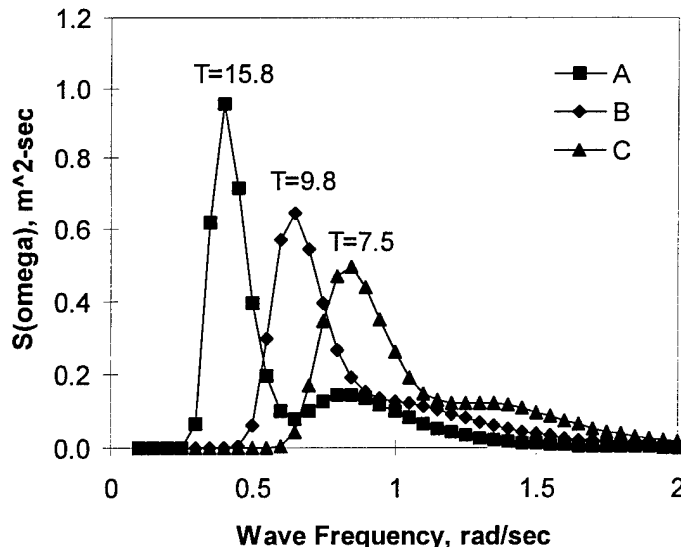


Figure 1. Three wave spectra with a significant wave height of 1.88 m (6.17 ft).

<sup>1</sup> A comprehensive discussion of wave spectra can be found in reference 1.

## Swells

A swell is generally taken to be uni-directional and to have a narrow banded spectrum which is often super-imposed on a wind-generated confused sea. Swell conditions are of particular interest because their uni-directionality and narrow-banded character can lead to large responses, depending on how the wave energy frequency band aligns with the motion responses of the floating body. Of course, spectra come in a variety of distributions and the swell component can represent any proportion of the total energy.

In Figure 1 the three spectra differ significantly, with the only shared characteristic being the total energy. The most obvious difference is with the values of  $T_0$ . In addition, the nature of the spectral shapes differs. Spectrum A is noticeably double-peaked, with a large portion of the energy concentrated at long periods. Spectrum A represents a swell superimposed on a seaway. For swells, a guideline is that the half-power band width is about 25 percent of the center frequency [2]. For example, for Spectrum A the peak of the energy spectrum occurs at  $\omega=0.4$ , with a value of  $S(\omega)$  of 0.95. Thus, by this measure, if Spectrum A has a swell component at  $\omega=0.4$ , when  $S(\omega) = 0.5 * 0.95 = 0.48$ , the frequency band should be approximately 0.2. Since it is approximately 0.15, this can be considered to be a swell component. However, neither Spectrum B nor C have swell components by this measure.

When considering such cases experimentally, the general approach is to select a period which is taken to be between 8 and 15 seconds on the west coast and somewhat shorter on the east coast of the United States which will excite a particular response which is typically roll for monohulls [2].

## **MOTION TRANSFER FUNCTIONS**

When it can be assumed that the responses of a ship are linear with respect to wave amplitude, the responses of a ship can be presented as transfer functions which are responses per unit wave amplitude. Some people refer to this as the response amplitude operator, while others refer to the square of this as the response amplitude operator. The units clarify what is being presented. Typically, surge, sway and heave are presented as response per wave amplitude, A, and pitch, roll and yaw are presented as response per wave slope,  $kA$ , where  $k = \omega^2/g$  is the wave number and  $g$  is the acceleration due to gravity. Transfer functions for heave and pitch in head seas and roll in beam seas are presented for one monohull with forward speed in Figure 2 and for a large floating platform in Figure 3. Two different wave headings have been chosen in order to highlight conditions of larger responsiveness. The character of the two sets of transfer functions is quite different, due to the significant differences in size and vehicle type. As is typical for monohulls, roll is lightly damped and heave and pitch are heavily damped. Roll for the monohull is lightly damped. The results for the large floating platform were derived from experiments where resolution at low frequencies is sometimes difficult.



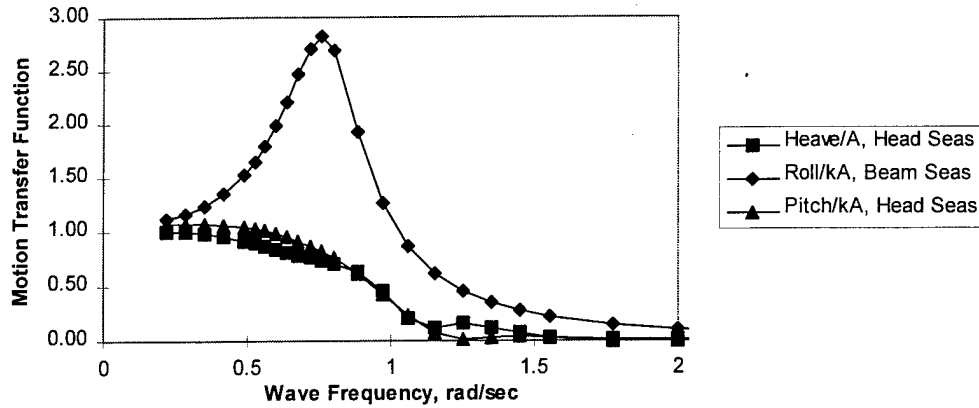


Figure 2. Heave, pitch and roll motion transfer functions of a monohull.

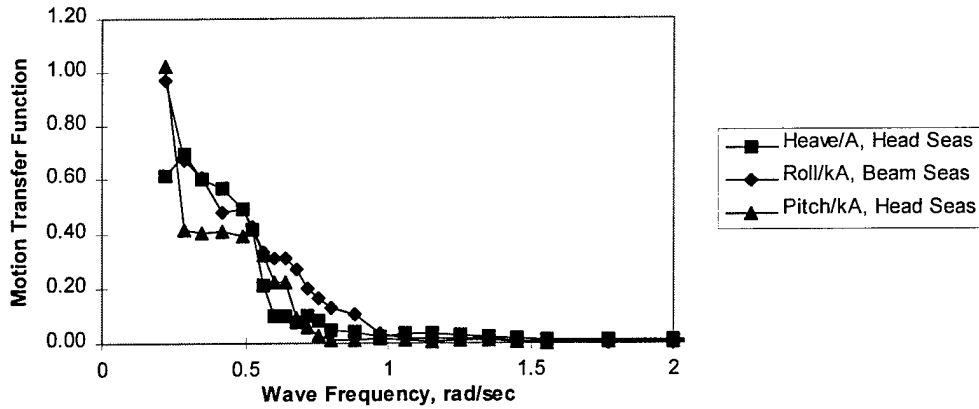


Figure 3. Heave, pitch and roll motion transfer functions of a large floating platform.

## RESPONSE SPECTRA

The wave environment provides excitation to any floating platform. This interaction is reflected in the response spectrum which is calculated by squaring a transfer function and multiplying by a wave spectrum. In order to obtain correct units, roll, pitch and yaw transfer functions, which have been non-dimensionalized in the manner described above, are multiplied by  $57.296 \cdot k$ . This removes the explicit dependence on wave frequency and changes the units from radians to degrees. In Figures 4 and 5, the transfer functions presented in Figures 2 and 3 have been modified in this manner. Heave is unchanged. These figures show that simply aligning transfer functions for angular responses with wave spectra can lead to a distorted expectation of the responsiveness of the hull form, since multiplying by the wave slope  $k$  reduces responses, most notably for low frequency (long period) waves.

It should be noted that the natural periods of responses are the periods associated with the wave frequency of encounter,  $\omega_e$ , of the peaks of the "modified" transfer function. The wave frequency and the wave frequency of encounter are related through the following expression:  $\omega_e = \omega + V k \cos \beta$ , where  $V$  is the body's forward speed and  $\beta$

is the relative wave heading,  $\beta = 0$  for head waves. When the floating body has no forward speed, the wave frequency and wave encounter frequency are equal.

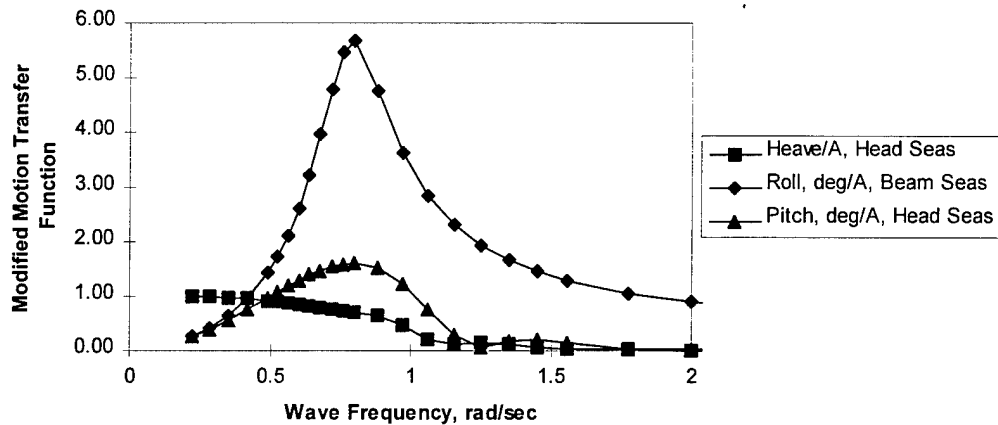


Figure 4. Heave and modified pitch and roll motion transfer functions of a monohull.

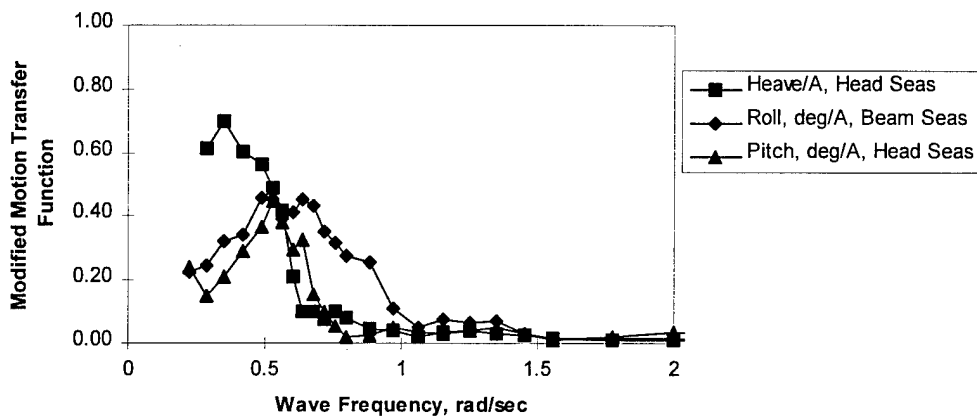


Figure 5. Heave and modified pitch and roll motion transfer functions of a large floating platform.

The effect of variations in wave spectra energy distribution given the same total energy is demonstrated in Figures 6, 7, and 8 for the monohull and Figures 9, 10 and 11 for the large floating platform. Each figure presents response spectra for heave, roll or pitch for the three spectra given in Figures 1 and 2. Clearly there are substantial variations.

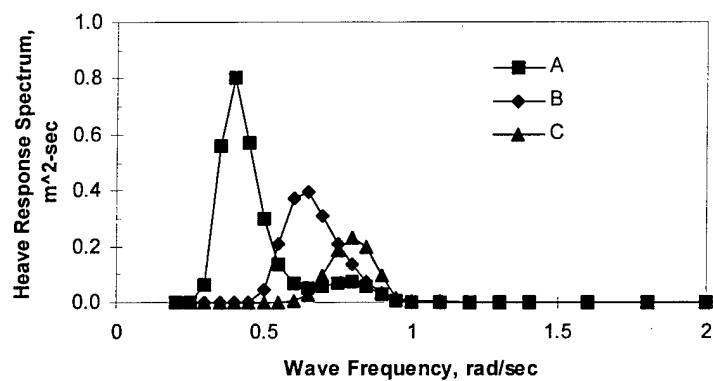


Figure 6. Heave response spectra of a monohull for Wave Spectra A, B and C.

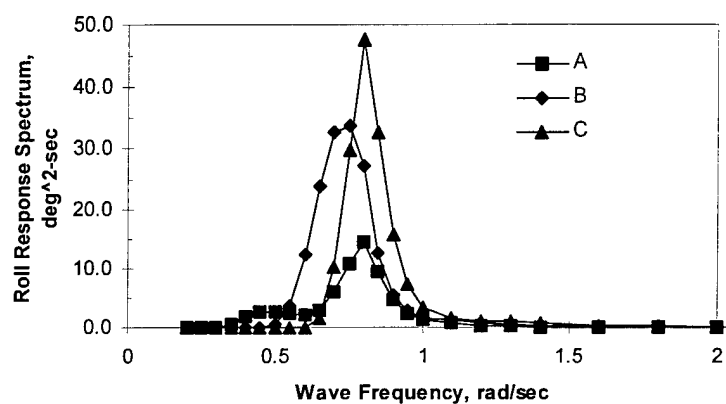


Figure 7. Roll response spectra of a monohull for Wave Spectra A, B and C.

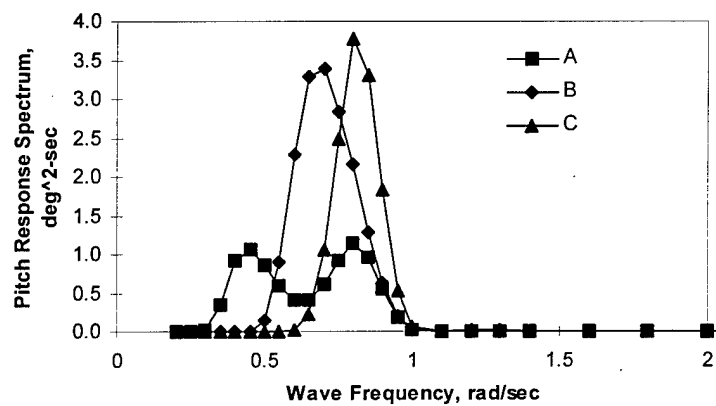


Figure 8. Pitch response spectra of a monohull for Wave Spectra A, B and C.

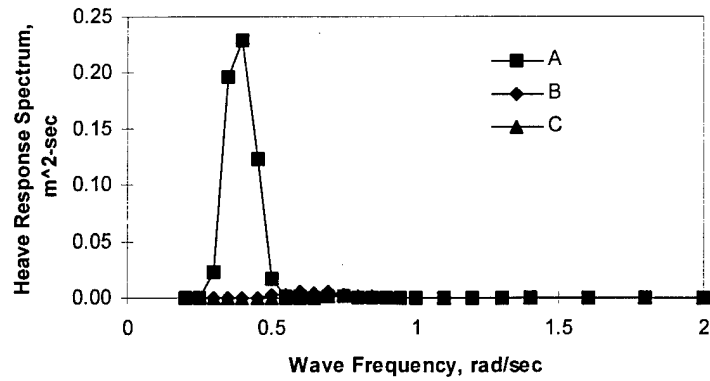


Figure 9. Heave response spectra of a large floating platform for Wave Spectra A,B and C.

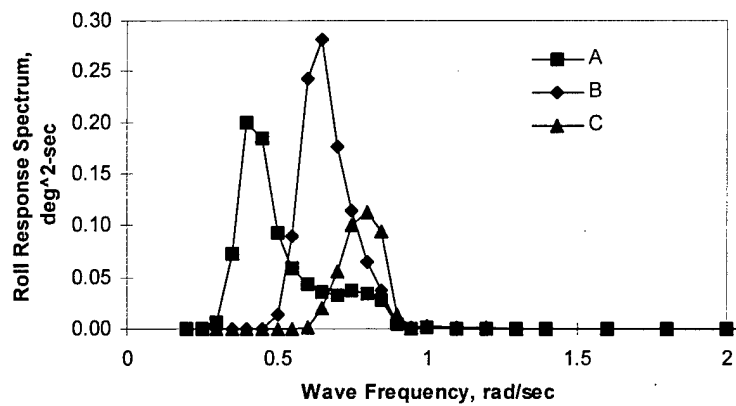


Figure 10. Roll response spectra of a large floating platform for Wave Spectra A, B and C.

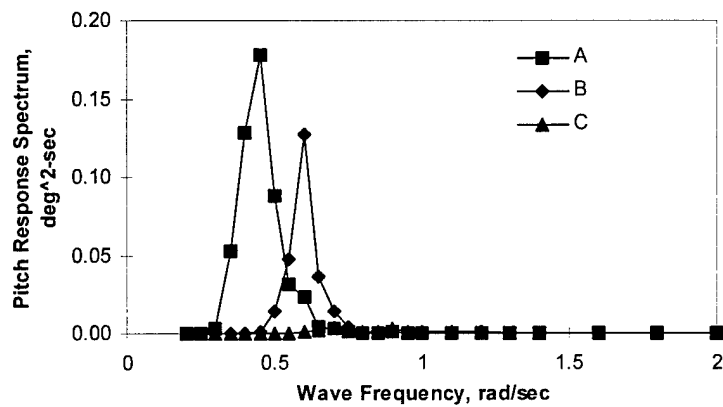


Figure 11. Pitch response spectra of a large floating platform for Wave Spectra A, B and C.

## SIGNIFICANT SINGLE AMPLITUDE RESPONSES

The measure often used for the responses of a floating body to waves is the root mean squared (RMS) of the time history of the response, or some multiple of it. If certain statistical assumptions are made, the RMS can be assumed to be equal to the square root of the integral of the response spectrum with respect to wave frequency. The significant single amplitude (SSA) is twice the RMS. The SSA is the average of the highest 1/3 of the amplitudes of the response. The SSA is often used in the statement of seakeeping motion criteria in Operational Requirements for a design. SSA for heave, pitch and roll for Spectra A, B and C are presented in Table 1.

Table 1. Significant single amplitude of motion responses of a monohull and a large floating platform to three spectra with a 1.88 meter significant wave height.

Spectrum	Monohull			Large Floating Platform		
	Heave, m (Head)	Roll, deg (Beam)	Pitch, deg (Head)	Heave, m (Head)	Roll, deg (Beam)	Pitch, deg (Head)
A( $T_0=15.8$ )	0.754	3.706	1.346	0.345	0.408	0.321
B ( $T_0=9.8$ )	0.602	5.738	1.856	0.071	0.455	0.223
C ( $T_0=7.5$ )	0.416	5.692	1.638	0.040	0.288	0.068

These results show that variations in wave spectral energy distribution and response spectra lead to variations in responses in some cases, but not in others, and that this variation is different for different platforms. Clearly, proper description of the wave environment in which a hullform is expected to operate is important.

## WAVE ENVIRONMENT

Compilations of observed and measured wave and wind data have been used in ship routing for some time. There are, of course, difficulties with consistency in observed and measured data and the data is sparse for many geographical locations. Therefore, mathematical models which describe the environment have been developed. In some applications, generalized information can be quite useful; however, in order to carry out predictions of seakeeping performance, details of the likelihood of conditions occurring and the wave spectrum must be known or assumed. In the discussion below, wave statistics and two wave spectral formulations which have been related to probabilities of occurrence will be discussed.

## SIX-PARAMETER WAVE SPECTRAL FORMULATION AND SPECTRAL FAMILY

Ochi and Hubble analyzed 800 spectra observed in the North Atlantic [3]. Based on this analysis, they developed a six parameter spectral formulation which models a range of spectra, including double peaked spectra and a variety of spectral shapes, and parameters which define a family of 11 spectra which cover the range of spectra which occur for a specified significant wave height.

The spectral formulation is:

$$S(\omega) = \frac{1}{4} \sum_{j=1}^2 \frac{\omega_{mj}^4}{\omega^5} \frac{\left( \frac{4\lambda_j + 1}{4} \omega_{mj}^4 \right)^{\lambda_j}}{\Gamma(\lambda_j)} \frac{H_{1/3j}^2}{\omega^{4\lambda_j + 1}} e^{-\left( \frac{4\lambda_j + 1}{4} \right) \left( \frac{\omega_{mj}}{\omega} \right)^4}$$

where  $\lambda_j$  is a shape parameter for the  $j^{\text{th}}$  spectrum and other quantities are as defined above. For the case where only one element in the summation is used ( $H_{1/32} = 0.0$ ) and  $\lambda$  is defined as 1.0, this formulation reduces to the Bretschneider spectral formulation which will be discussed below. The sharpness of the peak increases with increasing values of  $\lambda$ .

Based on their analysis, Ochi and Hubble developed a family of wave spectra which spans a 95% confidence band, with the most probable (MP) spectrum considered to occur 50% of the time and each of the remaining 10 considered to occur 5% of the time. The parameters for this family are given in Table 2. Note that  $\omega_{m1}$  is less than  $\omega_{m2}$ , so that the first term in the sum represents the lower frequency components of energy.

The resultant family of spectra for a mid Sea State 4 ( $H_{1/3} = 1.88$  m) is shown in Figure 12. The labels given in the left hand column of Table 2 correspond to those used in the plot legend. As shown in the figure, a wide range of energy distributions is modeled. Spectra A, B, and C in Figure 1 correspond to Spectra 4, MP, and 3 in Figure 12.

Table 2. Parameters of 6-parameter spectra for 95% confidence band (from reference 3).

	$H_{s1}$	$H_{s1}$	$\omega_{m1}$	$\omega_{m2}$	$\lambda_1$	$\lambda_1$
MP	$0.84 H_{1/3}$	$0.54 H_{1/3}$	$0.70 e^{-0.046H_{1/3}}$	$1.15 e^{-0.039H_{1/3}}$	3.0	$1.54 e^{-0.062H_{1/3}}$
1	$0.95 H_{1/3}$	$0.31 H_{1/3}$	$0.70 e^{-0.046H_{1/3}}$	$1.50 e^{-0.046H_{1/3}}$	1.35	$2.48 e^{-0.102H_{1/3}}$
2	$0.65 H_{1/3}$	$0.76 H_{1/3}$	$0.61 e^{-0.039H_{1/3}}$	$0.94 e^{-0.036H_{1/3}}$	4.95	$2.48 e^{-0.102H_{1/3}}$
3	$0.84 H_{1/3}$	$0.54 H_{1/3}$	$0.93 e^{-0.056H_{1/3}}$	$1.50 e^{-0.046H_{1/3}}$	3.00	$2.77 e^{-0.112H_{1/3}}$
4	$0.84 H_{1/3}$	$0.54 H_{1/3}$	$0.41 e^{-0.016H_{1/3}}$	$0.88 e^{-0.026H_{1/3}}$	2.55	$1.82 e^{-0.089H_{1/3}}$
5	$0.90 H_{1/3}$	$0.44 H_{1/3}$	$0.81 e^{-0.052H_{1/3}}$	$1.60 e^{-0.033H_{1/3}}$	1.80	$2.95 e^{-0.105H_{1/3}}$
6	$0.77 H_{1/3}$	$0.64 H_{1/3}$	$0.54 e^{-0.039H_{1/3}}$	0.61	4.50	$1.95 e^{-0.082H_{1/3}}$
7	$0.73 H_{1/3}$	$0.68 H_{1/3}$	$0.70 e^{-0.046H_{1/3}}$	$0.99 e^{-0.039H_{1/3}}$	6.40	$1.78 e^{-0.069H_{1/3}}$
8	$0.92 H_{1/3}$	$0.39 H_{1/3}$	$0.70 e^{-0.046H_{1/3}}$	$1.37 e^{-0.039H_{1/3}}$	0.70	$1.78 e^{-0.069H_{1/3}}$
9	$0.84 H_{1/3}$	$0.54 H_{1/3}$	$0.74 e^{-0.052H_{1/3}}$	$1.30 e^{-0.039H_{1/3}}$	2.65	$3.90 e^{-0.085H_{1/3}}$
10	$0.84 H_{1/3}$	$0.54 H_{1/3}$	$0.62 e^{-0.039H_{1/3}}$	$1.03 e^{-0.030H_{1/3}}$	2.60	$0.53 e^{-0.069H_{1/3}}$

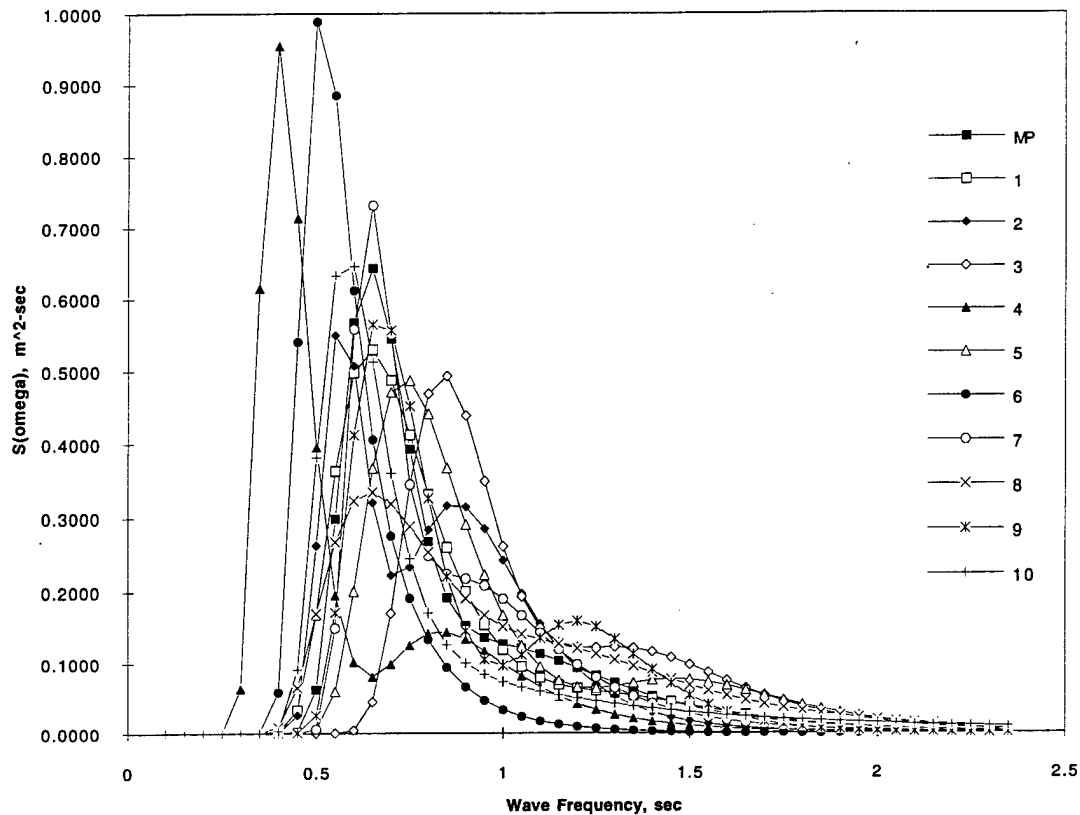


Figure 12. Ochi-Hubble 6-parameter spectral family for a mid Sea State 4.

## SOWM DATABASES

The Spectral Ocean Wave Model (SOWM) [4] is a hindcast mathematical model which utilizes wind data to generate spectra of ocean waves in deep water. The SOWM evolved into the Global Spectral Ocean Wave Model (GSOWM), and the Wave Model (WAM) which are more sophisticated and accurate mathematical models. The SOWM is limited to the Northern Hemisphere and does not include swell which has propagated from the Southern Hemisphere, while the GSOWM doesn't have these limitations. The GSOWM also includes modeling of nonlinearities related to wave interactions. The WAM model is more sophisticated and accurate than the GSOWM.

Ten years of SOWM-generated data was used to generate climatology databases for the North Atlantic, North Pacific, and Mediterranean Sea [5,6,7]. These databases include the joint probability of occurrence of significant wave height and spectral modal period for each month and for the year. Databases of joint probability of occurrence of significant wave height, modal period, and wind speed, which are also based on the SOWM database, are routinely utilized in evaluating US Navy ships using the Seakeeping Evaluation Program (SEP) [8]. An additional database was recently developed for several points in the Indian Ocean, based on nine years of GSOWM data.

These climatologies result from analyzing the mathematically generated spectra and determining the number of joint occurrences of characteristics of interest within

selected ranges. For example, for significant wave height-modal period tables, three feet (or greater) increments are used for the tables published in references 5, 6 and 7. (In the databases accessed by SEP, 0.5 meter increments are used.) The modal periods are the periods which correspond to the center frequency of 15 bands and range from 3.2 to 25.7 seconds. (They are: 3.24, 4.8, 6.32, 7.5, 8.57, 9.73, 10.91, 12.4, 13.85, 15.0, 16.4, 18.0, 20.0, 22.5, 25.7 seconds.)

Bales [9] used SOWM data from 13 points in the North Atlantic to develop a composite table of the range of likely spectra which is reproduced in Table 3. Subsequent to the development of this table, the SOWM database was significantly expanded; however, this table continues to serve as a reasonable guide. This table includes the relationship between sea state and significant wave height which is used by the US Navy's design community and others. Note that the "Most Probable Modal Period" column presents the modal period corresponding to the center frequency of the modal period band with the highest probability of occurrence and does not reflect anything about the distribution within that band. That is, the most probable modal period should not be rigidly applied.

Table 3. Sea State Table for the General North Atlantic (from Reference 9).

Sea State	Significant Wave Height (m) (ft)		Modal Period Range (sec)	Most Probable Modal Period (sec)	Sustained Wind Speed (kts)
0-1	0-0.1	0-0.33	-	-	0-6
2	0.1-0.5	0.33-1.64	3.3-12.8	7.5	7-10
3	0.5-1.25	1.64-4.10	5.0-14.8	7.5	11-16
4	1.25-2.5	4.10-8.20	6.1-15.2	8.8	17-21
5	2.5-4.0	8.20-13.12	8.3-15.5	9.7	22-27
6	4.0-6.0	13.12-19.69	9.8-16.2	12.4	28-47
7	6.0-9.0	19.69-29.53	11.8-18.5	15.0	48-55
8	9.0-14.0	29.53-45.93	14.2-18.6	16.4	56-63
>8	>14.0	>45.93	15.7-23.7	20.0	>63

The North Atlantic SOWM SEP database includes data for 57 geographical points, including the 13 used to develop Table 2. In Figure 13 the percent time of occurrence of spectra of various significant wave height bands is presented as a function of the center frequency spectral modal period for the composite of the 57 points. Various significant wave height bands were used in this figure, with more detail at the lower values. In Figure 14 similar information is presented for different sea state bands. These figures show that the range of likely modal periods narrows as significant wave height increases and that for some significant wave height bands, the most probable modal period is not much more likely than other modal periods.



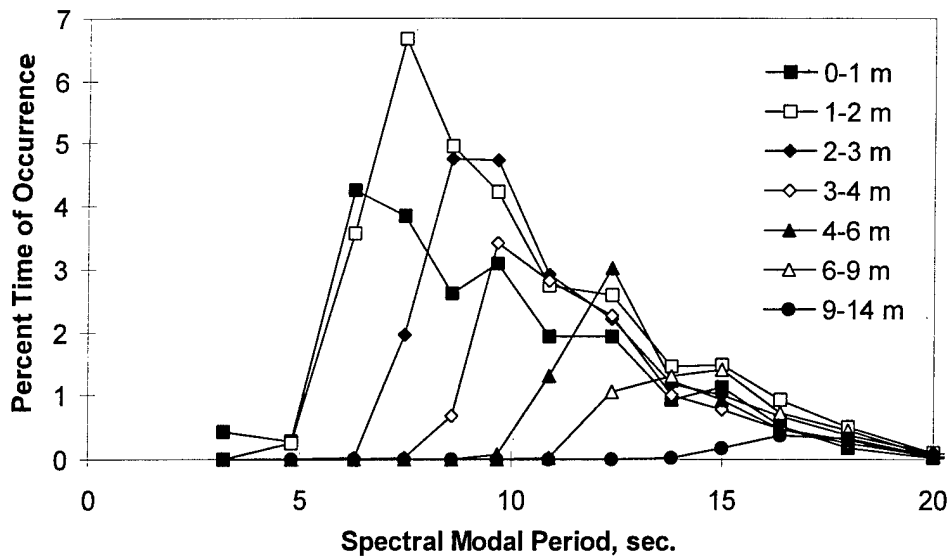


Figure 13. Percent time of occurrence of spectra in the North Atlantic as a function of spectral modal period for different significant wave height bands.

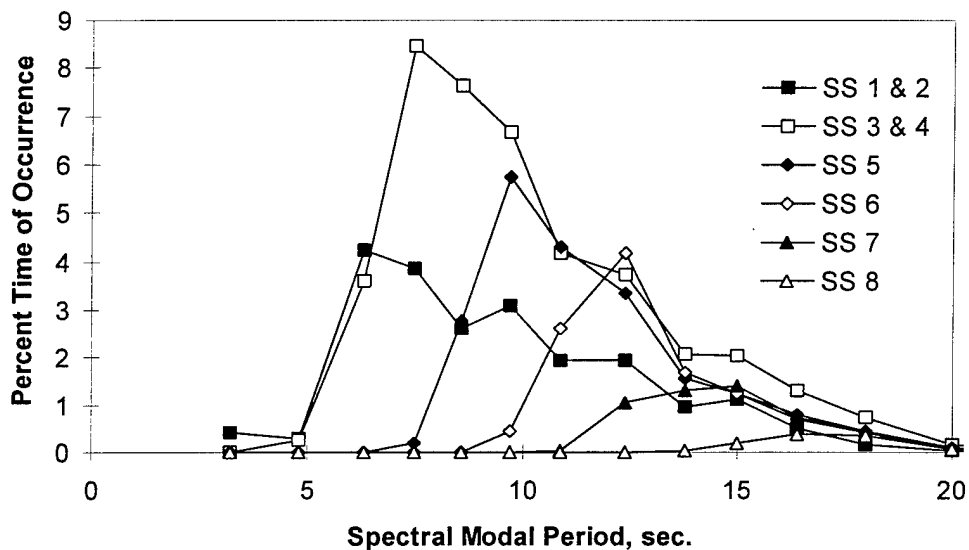


Figure 14. Percent time of occurrence of spectral modal period in the North Atlantic for different sea states.

### BRETSCHNEIDER SPECTRAL FORMULATION

The Bretschneider spectral formulation which is based on analysis of spectral data in the open ocean in the North Atlantic was recommended for use by the 15<sup>th</sup> International Towing Tank Conference (ITTC). The Bretschneider formulation is a function of both significant wave height and modal frequency and allows for representation of a wide range of single peaked wave spectra. Consequently, it is

convenient to use the Bretschneider wave spectral formulation in conjunction with the SOWM/GSOWM databases in seakeeping evaluations. The Bretschneider formulation is:

$$S(\omega) = \frac{1.25}{4} \frac{\omega_m^4}{\omega^5} H_{1/3}^2 e^{-1.25 \left( \frac{\omega_m}{\omega} \right)^4}$$

where  $S(\omega)$  is the spectral energy for the frequency  $\omega$  which is given in rad/sec. The units of  $S(\omega)$  depend on the units of  $H_{1/3}$ , so that they are  $m^2$ -sec when  $H_{1/3}$  is in meters.

$S(\omega)$  is plotted for a range of modal periods for 1.88 m (6.17 foot) significant wave height which is a mid Sea State 4 in Figure 15. The modal periods used are among those used in the SOWM tabulations. MP is used to indicate the most probable modal period for this sea state, according to the information in Table 3.

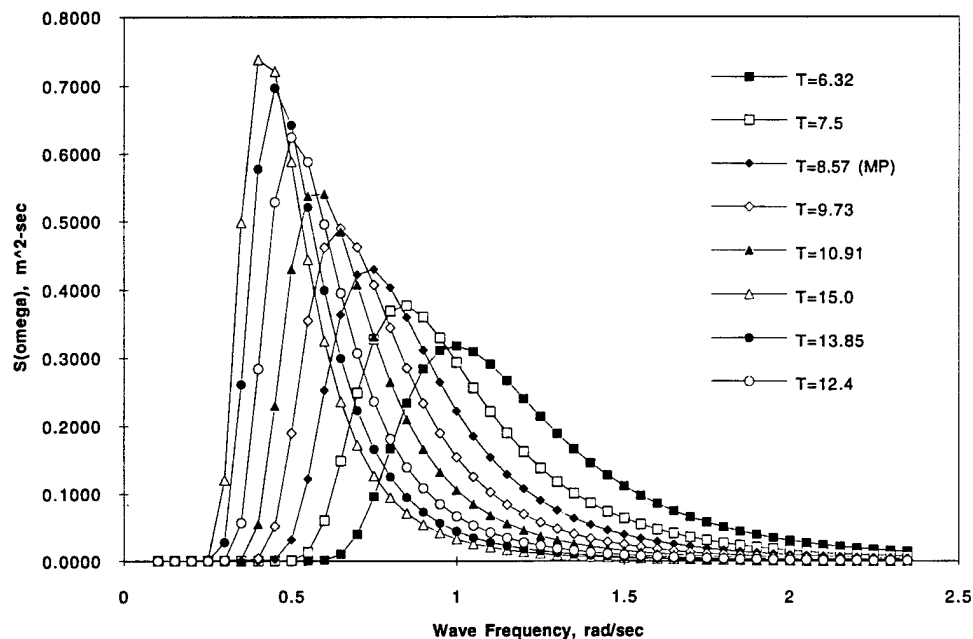


Figure 15. A range of Bretschneider spectra for a mid Sea State 4.

## OTHER SPECTRAL FORMULATIONS

Other idealized spectral formulations have been developed. These include the Pierson-Moskowitz spectrum which represents fully developed seas and is a special case of the Bretschneider formulation and the JONSWAP (Joint North Sea Wave Project) spectrum which was developed based on data for fetch limited conditions. The JONSWAP spectral formulation includes a frequency-dependent multiplicative factor which affects the character of the peak which is equivalent to a Bretschneider spectrum when the peak factor is 1.0 [1]. The JONSWAP spectrum can be used to represent a variety of spectra, including swells, by increasing the value of the peak factor. These spectra represent only single-peaked spectra and will not be included in examples here.

## WAVE DIRECTIONALITY

In addition to spectral shape, the manner in which the wave energy is spread as a function of direction is of interest. Analytical studies often include both uni-directional and multi-directional seas. Responses to multi-directional seas are typically calculated using the cosine-squared spreading function, although other spreading functions are in development. Since most experimental facilities cannot generate multi-directional seas, consideration is typically limited to uni-directional seas. However, since the most severe vertical plane responses generally occur in uni-directional head seas and the most severe lateral plane responses generally occur in uni-directional beam seas to seas, this approach is useful for evaluating designs.

Of course, since swells are essentially uni-directional, it is appropriate to represent a single peaked spectrum which represents a swell as being uni-directional. However, for double peaked spectra which include a swell component, such as are represented by some Ochi-Hubble 6-parameter spectra, it would be more appropriate to apply different directionality distributions to the two components. That is, the swell component could be represented as uni-directional and a spreading function such as the cosine-squared function could be applied to the other component, or the two components could be represented as coming from different directions.

## SELECTION OF WAVE SPECTRA FOR INVESTIGATIONS

When experimental studies are undertaken, only a limited number of wave spectra can be considered due to time and cost constraints. As discussed above, when selecting a limited number of wave spectra for a seakeeping study, consideration should first be given to what wave conditions might occur, and then to which of those likely wave conditions will excite large responses in the floating platform. If an operational region has been specified, climatologies such as those developed based on the SOWM/GSOWM data can be used. When a region has not been identified, it is customary to utilize statistics available for the General North Atlantic. The Ochi-Hubble analysis is applicable to this region and summary statistics for this region have been developed based on the SOWM climatology, such as those presented in Table 3. When a reasonably accurate means of carrying out frequency domain computer simulations exists, it should be used to predict responses to a range of wave spectra. Such a study may be the final product or may be used to provide guidance in selecting a limited number of spectra. In the absence of such a prediction capability, motion natural periods may be estimated.

When operation is not geographically limited and swells are of interest, for a specified significant wave height, three spectra are suggested:

- 1) A Bretschneider spectrum with a highly probable modal period. This spectrum can be based on the SOWM data for the intended operational region or the General North Atlantic if no operational region is identified. Table 3 and Figures 3 and 4 can be used to identify the highly probable modal period. As noted above, the "most probable" modal periods in Table 3 should not be rigidly applied since the values listed are simply the center frequency of a range of modal frequencies and may not be significantly more likely than other periods within a broad range of modal periods. Thus the statistics are really applicable to a range of values, rather than just the one listed for a

significant wave height band. Sometimes response characteristics of the floating vehicle are used to select a highly probable modal period which will excite large responses.

2) The 6-parameter spectrum identified as most probable. Such a spectrum may be double-peaked, but it is more likely that the shape of the spectrum which results from summing two spectra will have a more subdued character. Spectrum B in Figure 1 is the spectrum identified as most probable. Note that the spectrum identified as most probable is based on data for the North Atlantic.

3) A 6-parameter spectrum which represents a swell superimposed on a seaway. This type of spectrum is of particular interest when a response is lightly damped, as with the case of roll for monohulls. Also, large floating bodies may have long natural periods so that other responses such as heave may have large responses, as seen for Spectrum A in the example given above. This type of spectrum may be included among the 6-parameter family of 11 spectra, depending on the significant wave height. If evaluation of large motion responses is a goal and one of the spectra in the family has a peak near a response natural period, that spectrum can be selected. Alternatively, the  $\omega_{m1}$  associated with a peak of one of the 11 6-parameter spectra can be modified somewhat in order to match a frequency associated with a response natural period.

In Figure 16, four spectra for a mid sea state 4 are presented. The first two correspond to those suggested in 1) and 2) above and the other two are alternative suggestions for the third spectrum, as described in 3) for the case where the natural period of the response of interest is about 14.0 seconds. The first of the alternatives is the Spectrum A in Figure 1 which is also the spectrum labelled "4" in the 6-parameter family (see Table 2). It has a peak which among the 11 spectra in the family is closest to the period of interest. The other spectrum results from a modification of spectrum "4" with  $\omega_{m1}$  modified. In general, selection of either of the two latter spectra is reasonable since the spectra in the 6-parameter family represent a range of likely spectra.

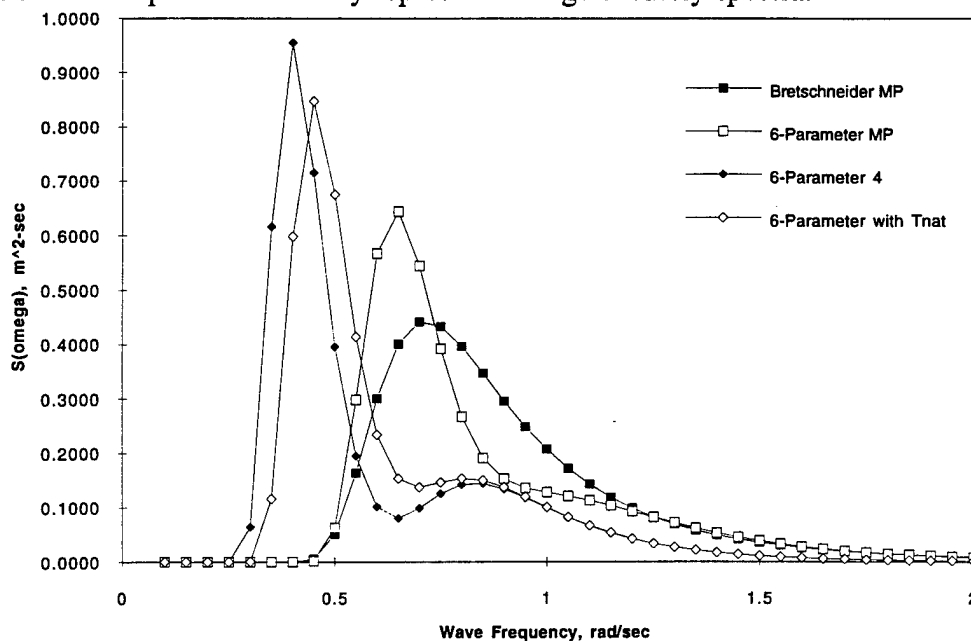


Figure 16. A selection of wave spectra for design evaluation with a significant wave height of 1.88m (6.17 feet).

## CONCLUSIONS

A method for selecting wave spectra for consideration in seakeeping design evaluations has been presented. Included are single and double peaked spectra, including ones with swell components. The interrelationships among ship motions, wave spectra, and response characteristics has been discussed through use of examples. In selecting a small number of wave spectra which are likely to occur, data relating sea state and spectral modal periods is utilized in conjunction with the Ochi-Hubble 6-parameter wave spectral family. Both of these should be used in a non-rigid way, since each specified wave spectrum in fact represents a range of spectra. The objective of a project must be considered before wave spectra are selected. In some design evaluations, "worst" cases which excite large motion responses are sought, while in others "typical" cases are of interest. It is impossible to select a perfect spectrum for either of these objectives, but the goal will affect the strategy applied in selecting a small number of wave spectra.

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